CHAPTER 8- CIRCULAR MEASURE

8.1 Radian

1. In lower secondary, we have learned the unit for angle is degree. In this chapter, we will learn one more unit for angle that is radian.

2. When the value of the angle 1 radian, then the length of the arc is equal to the length of the radius.

3. From this information, we can deduce that:

\[
\frac{1 \text{ rad}}{360} = \frac{r}{2\pi}
\]

\[
1 \text{ rad} = \frac{r}{2\pi} \times 360
\]

\[
2\pi \text{ rad} = 360^\circ
\]

4. \( \pi \text{ rad} = 180^\circ \)

\[
1 \text{ rad} = \frac{180^\circ}{\pi}
\]

\[
= 57.3^\circ \text{ or } 57^\circ 18'
\]

5. \( 2\pi \text{ rad} = 360^\circ \)

\[
1^\circ = \frac{\pi}{180} \text{ radian}
\]

8.1.1 Converting Measurements in degree to radian

Example 1:

Convert 120° to radians

Solution:

\[
1^\circ = \frac{\pi}{180} \text{ radian}
\]
120° = 120 × \( \frac{\pi}{180} \) radian

\[ = \frac{2\pi}{3} \text{ radian or } 2.0947 \text{ radian} \]

**Example 2:**
Convert 112°36' to radians

**Solution:**

\[
112^\circ 36' = 112^\circ + 36'
\]

\[
= 112^\circ + \left( \frac{36}{60} \right)^\circ
\]

\[
= 112^\circ + 0.6^\circ
\]

\[
= 112.6^\circ
\]

\[
1^\circ = \frac{\pi}{180} \text{ radian}
\]

\[
112.6^\circ = 112.6 \times \frac{\pi}{180} \text{ radian}
\]

\[= 1.965 \text{ rad} \]

8.1.2 Converting Measurements in radian to degree

**Example 1:**
Convert \( \frac{\pi}{6} \text{ rad} \) to degree

**Solution:**

\[
1^\circ \text{ rad} = \frac{180^\circ}{\pi}
\]

\[
\frac{\pi}{6} \text{ rad} = \frac{\pi}{6} \times \frac{180^\circ}{\pi}
\]

\[= 30^\circ \]

**Example 2:**
Convert 1.36 rad to degree

**Solution:**

\[
1^\circ \text{ rad} = \frac{180^\circ}{\pi}
\]
1.36 rad = \( 1.36 \times \frac{180^\circ}{\pi} \)

\[
= \frac{244}{\pi}^\circ
\]

\[
= 77.92^\circ
\]

**EXERCISE 8.1**

1. Convert each of the following values to degrees and the nearest minute. \((\pi = 3.142)\)

(a) 0.37 rad

(b) 2.04 rad

(c) 1.19 rad

2. Convert each of the following values to radians, giving your answer correct to 4 significant figures. \((\pi = 3.142)\)

(a) \(248^\circ9^\prime\)

(b) \(304^\circ22^\prime\)

(c) \(46^\circ14^\prime\)

**8.2 LENGTH OF ARC OF A CIRCLE**

If the unit of angle is degrees, The angle of a whole turn is \(360^\circ\).

\[
\frac{S}{\text{Circumference of a circle}} = \frac{\theta}{\text{angle of a whole turn}}
\]

We know that \(2\pi \text{ rad} = 360^\circ\)

If the unit of the angle is in radians, The angle of a whole turn is \(2\pi \text{ rad}\).
Hence,

\[ \frac{S}{\text{Circumference of a circle}} = \frac{\theta}{2\pi} \]

\[ \frac{S}{2\pi r} = \frac{\theta}{2\pi} \]

\[ S = \frac{2\pi \theta}{2\pi} \]

\[ S = r\theta \]

\[ \theta \text{ is in radians.} \]

**Example 1:**

The diagram above shows a circle with a sector $POQ$ and radius 6 cm. Given the length of minor arc $PQ$ is 7.68 cm. Find the value of $\theta$, in radians.

**Solution:**

The formula of length of arc if angle in radians is

\[ S = r\theta \]

Given $r = 6$ cm and $S = 7.68$ cm,

\[ 7.68 = 6\theta \]

\[ \theta = 1.28 \text{ rad} \]

**Example 2:**

Given a circle with centre $O$ and radius 5 cm. Find the length of arc $PQR$ if the angle $\angle POR$ is 1.2 rad.

**Solution:**

The formula of length of arc if angle in radians is

\[ S = r\theta \]

Given $r = 5$ cm and $\theta = 1.2$ rad,

\[ S = (5)(1.2) \]

\[ S = 6 \text{ cm} \]
**EXERCISE 8.2**

1. In the diagram 3, the perimeter of sector OPQ is 32 cm.

   ![Diagram 3](image)

   (a) Express \( r \) in terms of \( \theta \).
   
   (b) Find the value of \( r \) if \( \theta = 1.2 \text{ rad} \).

2. The angle subtended at the centre by an arc ABC with a radius 4.2 cm is 1.4 radians. Find the length of arc AB.

3. The length of a minor arc of a circle is \( 2\pi \text{ cm} \). The angle subtended at the centre of by the major arc is \( 240^\circ \). Find the radius of the circle.

4. Diagram 4 shows two arcs AB and CD with a common centre O. It is given that BD = AC = 3 cm.

   ![Diagram 4](image)

   If the perimeter of the shaded region ABCD is 12 cm, find the length of radius OB.

**8.3 AREA OF SECTOR OF A CIRCLE**

\[
\text{Area of sector} = \frac{\theta}{2\pi} \times \pi r^2
\]

\[
= \frac{\theta}{2} \times r^2
\]
If the unit of angle is degrees,
The angle of a whole turn is $360^\circ$.

\[
\frac{A}{\pi r^2} = \frac{\theta}{360^\circ}
\]

\[
A = \frac{\theta}{2\pi} \times \pi r^2
\]

$\theta$ is in degrees.

We know that $2\pi \text{ rad} = 360^\circ$

If the unit of the angle is in radians,
The angle of a whole turn is $2\pi$ radian.

Hence,

\[
\frac{A}{\pi r^2} = \frac{\theta}{2\pi}
\]

\[
A = \frac{\pi r^2 \theta}{2\pi}
\]

\[
A = \frac{1}{2} r^2 \theta
\]

$\theta$ is in radians.

Besides that,

\[
A = \frac{1}{2} r^2 \theta
\]

\[
= \frac{1}{2} r.r.\theta
\]

\[
A = \frac{1}{2} r S
\]

Both formula can be used depends on the information given in the question.

**Area Of Triangle**

![Diagram of a triangle with base 'b', height 'h', and sides 'a' and 'c'.]

The formula for area of triangle that is $\frac{1}{2} \times \text{base} \times \text{height}$ can only be used in situation where there is right angle triangle. In the situations that the triangle is not a right-angled triangle, we cannot use the formula.
\[ \sin A = \frac{h}{AB} \]
\[ h = AB \sin A \]

Substitute 2 into 1,

Area of segments
The formula is

\[ \text{Area} = \frac{1}{2} \times AC \times AB \times \sin A \]
\[ \text{Area} = \frac{1}{2} \times AB \times BC \times \sin B \]
\[ \text{Area} = \frac{1}{2} \times AC \times BC \times \sin C \]

This formula can be used to find the area of all types of triangle as long as there is enough information given. The sine of an angle is multiplied by the length of line that joining to form the angle. For example, sine A is multiply by AB and AC that are the lines that joining to form the angle A.

**Example 1:**

The diagram above shows a circle with a sector POQ and radius 6 cm. Given the length of minor arc PQ is 7.68 cm. Find the value of \( \theta \), in radians. Hence, find the area of the shaded region.

**Solution:**
The formula of length of arc if angle in radians is

\[ S = r \theta \]

Given \( r = 6 \) cm and \( S= 7.68 \) cm,

\[ 7.68 = 6 \theta \]
\[ \theta = 1.28 \text{ rad} \]
Area of shaded region = \( \frac{1}{2}(6^2)(1.28) - \frac{1}{2}(6^2)(\sin 1.28) \)
= 23.04 - 17.24
= 5.8 \text{ cm}^2

**Example 2:**
The diagram shows a sector POQ with centre O and radius 16 cm. Point R lies on OP such that OR: OP = 5:8. Given that \( \angle ORQ = 90^\circ \)

Calculate
(a) the value of \( x \), in radians.
(b) the area of shaded region, in \( \text{cm}^2 \)

**Solution:**
Given \( \frac{OP}{OR} = \frac{5}{8} \) and \( OR = 16 \text{ cm} \).
Substitute \( OR = 16 \text{ cm} \) into \( \frac{OP}{OR} = \frac{5}{8} \).

\[
\frac{OP}{16} = \frac{5}{8} \Rightarrow OP = \frac{5}{8} \times 16 \Rightarrow OP = 10 \text{ cm}
\]

a) Given that \( \angle ORQ = 90^\circ \)
\[
\cos x = \frac{10}{16} \Rightarrow x = \cos^{-1}\left(\frac{10}{16}\right) = 0.8957 \text{ rad}
\]
b) Area of shaded region = \( \frac{1}{2} (16^2)(0.8957) - \frac{1}{2} (10)(6)(\sin 0.8957) \)
\[ = 114.6496 - 62.4517 \]
\[ = 52.1979 \text{ cm}^2 \]

Use the formula for the area of triangle.

**EXERCISE 8.3**

1. In the diagram 4, the area of sector OPQ is 21.6\( \text{ cm}^2 \).

(a) Express \( r \) in terms of \( \theta \).
(b) Find the value of \( r \) if \( \theta = 1.2 \) rad.

2. The angle subtended at the centre by an arc ABC with a radius 3.6 cm is 1.15 radians. Find the area of arc AB.

3. The area of a sector of a circle is \( \pi 12 \text{ cm}^2 \). The angle subtended at the centre of by the sector is 270°. Find the radius of the circle.

**8.4 SOLVING PROBLEMS INVOLVING RADIANS**

**Example:**

The diagram above shows a semicircle ABCDE with centre P and rhombus PBQD. Calculate:
(a) the radius of semicircle ABCDE
(b) the angle of \( \theta \) in radian
(c) the area of sector PBCD
(d) the area of shaded region
Solution:
(a) \[ PE^2 = (12 - 8)^2 + (4 - 4)^2 \]
\[ PE = \sqrt{16} \]
\[ PE = 4 \]
Radius = 4 unit

(b) \[ \cos \angle PBX = \frac{1}{4} \]
\[ \angle PBX = \cos^{-1}\left(\frac{1}{4}\right) \]
\[ = 1.1181 \text{ rad} \]
\[ \theta = 1.3181 \times 2 \]
\[ = 2.6362 \text{ rad} \]

(c) Area of sector PBCD = \[ \frac{1}{2} (4^2)(2.6362) \]
\[ = 21.0896 \text{ unit}^2 \]

(d) Area of shaded region = \[ 21.0896 - 2 \left( \frac{1}{2} (4)(4)(\sin 2.6362) \right) \]
\[ = 21.0986 - 7.7464 \]
\[ = 13.3442 \text{ cm}^2 \]

CHAPTER REVIEW EXERCISE
1. Diagram below shows a sector OPQ with centre O. and radius 4 cm.

Given that the perimeter of the sector is 20 cm, find the angle of the sector in radians.
2. The perimeter and area of a sector of a circle are 19 cm and 22.5 cm\(^2\) respectively. Calculate the possible values of the radius of the sector and the angle of the sector.

3. A sector has an area of L cm\(^2\) and a radius of 4.5 cm. Given that angle subtended at the centre of the circle is 135°, find the value of L. \(\pi = 3.142\)

4. The angle subtended at the centre of a circle by a minor arc is 40°. Find the ratio of length of the minor arc PQ to the length of the major arc PQ. \(\pi = 3.142\)

5. In diagram below, O is the centre of the arcs PQ and RS. The perimeter of the figure is 30 cm.

(a) Express α in terms of β
(b) Find the values α and β in radians, if the area of sector OPQ is 10 cm\(^2\)

6. The diagram below shows a sector OPQ constructed by bending a wire of length 40 cm.

Calculate
(a) the value of r
(b) the area of sector OPQ